Polarizing frequency of a fluid plasma antenna element

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1 Introduction

Plasma antennas are a new concept in adaptive antennas. They use the properties of low-energy plasma to allow rapid adjustments of their optimal operating frequency. Some of the earliest plasma antenna concepts appear in patents by Anderson, et. al. [1] [2] [4] Experimental verification of plasma antenna properties has been done in [5], where they also look at antenna efficiency, and in [8] using an exotic means of generating the plasma.

This paper uses the fluid plasma model developed in [6], and presents an expression for the current distribution of a plasma antenna element. It then makes some comments regarding the frequencies at which the antenna element is properly polarized. Although the results are valid specifically for an element in the shape of a prism, the general solution procedure will work for other geometries.

2 Basic Equations – Maxwell & Navier-Stokes

The plasma antenna model that will be described is that of a prism filled with a fluid plasma (the plasma tube). The fluid plasma model idealizes the aggregate motions of individual particles as the motion of a continuous, charged fluid. The motion of this charged fluid is identified with the electric current density, which acts as a forcing term for electromagnetic phenomena in the usual way. Similarly, electromagnetic fields can apply forces to the charged fluid by way of the Lorentz force. [7] It is useful to consider small amplitude, sinosoidal waves; even relatively high powered RF applications do not induce nonlinear plasma effects. This results in a linearized, frequency-dependent system of Maxwell equations:

$$\nabla \times \mathbf{E} = i\omega\mu\mathbf{H}, \tag{1}$$

$$\nabla \times \mathbf{H} = \sum_j N_j q_j \mathbf{u}_j - i\omega\epsilon\mathbf{E}, \tag{2}$$

$$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon}\sum_j n_j q_j, \text{ and } \nabla \cdot \mathbf{H} = 0. \tag{3}$$

For each type of charged particle j, the following fluid equations hold:

$$-i\omega n_j + N_j \nabla \cdot \mathbf{u}_j = 0, \tag{4}$$

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and

$$-i\omega N_j \mathbf{u}_j + c_j^2 \nabla n_j - \frac{N_j q_j}{m_j}\mathbf{E} = 0. \tag{5}$$

In the above, ω represents the angular frequency of the waves. N_j represents the bulk density of the charged particles, n_j is perturbed particle density, $\mathbf{u}_j$ represents the fluid velocity, q_j represents the charge of a single particle of type j, likewise m_j represents the mass of a single charged particle, and c_j represents their thermal acoustic speeds.

3 Velocity elimination

The system of equations describing a fluid plasma can be simplified by eliminating the fluid velocity terms. This is done by solving (5) for $\mathbf{u}_j$, which yields

$$\mathbf{u}_j = \frac{iq_j}{\omega m_j}\mathbf{E} - \frac{ic_j^2}{\omega N_j}\nabla n_j. \tag{6}$$

This can be used to substitute for the velocity terms appearing in (2) and (4). Combining Faraday's Law and the Ampère-Maxwell Law in the usual way, a new system is obtained:

$$\nabla^2 \mathbf{E} + \left(\omega^2 - \sum_j \omega_{pj}^2\right)\mu\epsilon\mathbf{E} = \frac{1}{\epsilon}\sum_j \left(1 - c_j^2\mu\epsilon\right) q_j \nabla n_j \tag{7}$$

with for each type of charged particle, j

$$\nabla^2 n_j + \left(\frac{\omega^2 - \omega_{pj}^2}{c_j^2}\right) n_j = \frac{\omega_{pj}^2}{c_j^2 q_j}\sum_{k\neq j} n_k q_k. \tag{8}$$

(The ω_{pj} terms represent the "plasma frequencies" of each charged particle type, and are defined as in [7] as

$$\omega_{pj}^2 = \frac{N_j q_j^2}{\epsilon m_j}.) \tag{9}$$

Notice that in this new formulation, the forcing terms for the acoustic waves are entirely acoustic, while the forcing for the electromagnetic waves is also entirely acoustic. In the absense of a boundary, electromagnetic waves will not induce acoustic waves. However, acoustic waves will be accompanied by a wave-like electric field. [6] The introduction of a boundary allows the transfer of energy betweeen both types of waves.

4 Rigid boundary condition for each type of particle

The equation for the fluid velocity (6) points to a natural boundary condition to impose along the inside of the plasma tube. Acoustic waves inside the tube remain inside, and cannot couple into acoustic waves outside the tube. This restricts the normal component of the fluid velocity to be zero on the boundary of the tube, and yields

$$d\mathbf{S} \cdot \mathbf{E} = \frac{c_j^2 m_j}{N_j q_j} d\mathbf{S} \cdot \nabla n_j \text{ (on the tube surface).} \tag{10}$$

Looking at the form of (10), it is clear that acoustic and electromagnetic waves can transfer energy along the boundary.

5 One-component Simplification

The plasma described in the previous sections consists of an arbitrary number of charged particle types. Typically, the plasmas used in an antenna will contain only two types: electrons and ions. The ions are more massive than the electrons ($m_{ion} \gg m_{electron}$), but have opposite charge ($q_{ion} = -q_{electron}$). Since plasmas are quasineutral, there are roughly the same number of electrons as ions ($N_{ion} = N_{electron}$). [7] If $c_{ion} \approx c_{electron}$ in (10), then it is clear that $|n_{ion}| \ll |n_{electron}|$. This means that the electron-related effects of the plasma will dominate, and it is safe to take $n_{ion} \approx 0$ in (7), (8), and to neglect it altogether in (10).

6 Solution

Consider the case of a right prism-shaped plasma tube. That is, consider a region $\{(x, y, z)|0 < z < L \text{ and } (x, y) \in R \subset \mathbb{R}^2\}$. In order to generate the plasma, typically high-voltage electrodes must be placed at either end of the tube. (See [3] or [8] for other techniques for generating plasma for antennas.) These electrodes will be represented by placing conductive boundaries at $z = 0$ and $z = L$. Away from these conductors, (8) can be solved to yield solutions in the form of sums of terms like

$$n_{electron} = W(x, y) \cos\left(\frac{p\pi}{L} z\right), \text{ where } p \in \mathbb{Z}. \tag{11}$$

In simple geometries, (such as a rectangular prism) it is easy to solve for the electric field in the plasma by observing that $\nabla n_{electron}$ provides the forcing for the electric field. This splits the electric field solution into a particular solution (from the forcing terms) and a homogeneous solution (to satisfy (10)). Observe that there are particular frequencies where $W(x, y)$ is a harmonic function on R. These occur when

$$\omega^2 = \frac{p^2 \pi^2 c_{electron}^2}{L^2} + \omega_{p,electron}^2. \tag{12}$$

Further, if there is no externally applied electric field, then $W(x, y) = \text{const}$. Then, it is apparent that the forcing term in (7) drives the electric field sinosoidally in the z-direction only, and that (10) removes any normal component of the electric field on the boundary. The plasma tube has become an antenna polarized in the z-direction.

7 Discussion

From (12), the plasma antenna is tunable. That is, it behaves much like a conventional antenna near a discrete set of frequencies. These frequencies can be adjusted by changing the thermal acoustic speed ($c_{electron}$) and the plasma frequency ($\omega_{p,electron}$).

Away from these polarizing frequencies, the specific distribution of charges and electric field will depend on the geometry of the tube. The charge distribution will no longer be uniform in the $z = \text{const}$ planes. This will induce some component of the electric field normal to the boundary. The antenna will not be polarized, and therefore loses some radiating efficiency.

8 Future Work

Although this paper describes how fields and charges inside a plasma antenna behave, little theoretical work has been done to describe the radiation pattern of fluid plasma antennas. Clearly, when (12) is satisfied, this reduces to a relatively standard antenna calculation. However, the off-frequency patterns have not been extensively studied.

The condition $c_{ion} \approx c_{electron}$ eliminates the ion interactions in the leading order approximations, but it is conceivable that this will not hold in general. Removing this condition requires one to solve the full system of equations, and is more complicated.

References

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